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LETTER TO THE EDITOR

Geometrical and electrical properties of crumpled wires*

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Abstract. Geometrical and electrical properties of crumpled wires are investigated. Power laws connecting the geometrical and electrical properties of these random systems are studied and critical exponents are calculated. In particular we have obtained the resistance exponent ζ which seems to be compatible with a previous conjecture.

Geometrical and physical properties of crumpled surfaces and strips (CS) obtained from random and irreversible compaction of paper sheets and aluminium foils have been recently studied [1–3]. Besides representing an interesting problem in surface statistics on their own, CS can be of potential interest in polymer and surface physics as well as in biological problems [4]. Thus, CS can be e.g. a simplified folding model to study the configurational properties of two-dimensional polymers or membranes containing in their interiors many tightly packed apolar groups shielded from an exterior solvent [4].

This letter deals with the geometry and with the electrical resistance of crumpled wires (CW). Here we present results for non-equilibrium configurations of crumpled wires of length L and diameter ρ with $50 \leqslant L/\rho \leqslant 60\,000$. In our experiment 154 wires, with length L varying from 5 to 3000 cm, were manually crumpled into approximately spherically compact balls (in order to obtain a high concentration of lateral contacts between different regions of the wire) and divided into three groups $G(\rho)$ with ρ standing for the diameter of the three types of wires used, namely $\rho = 0.1$, 0.08 and 0.05 cm. The CW balls so obtained are random and irreversible structures. Although we did not specify the force with which the CW were compressed, the compression process is very nonlinear and beyond a certain compression the diameter of the CW balls does not change appreciably. The electrical resistance of the wires (made of distinct compositions of lead and tin) was 0.23 ± 0.03, 0.98 ± 0.03 and $0.934 \pm 0.005\ \Omega/\mathrm{m}$, respectively. Each $G(\rho)$ was formed by seven equivalent families of CW with $n(\rho)$ different values of L. Thus $n = 8$ for $\rho = 0.1$ and 0.08 cm (corresponding to $L = 5$, 10, 20, 40, 100, 300, 1000 and 3000 cm) and $n = 6$ for $\rho = 0.05$ cm (corresponding to $L = 20$, 40, 100, 300, 1000 and 3000 cm). From each one of these 154 CW we defined an average diameter as the arithmetic mean of seven measurements of the diameter along seven directions taken at random. Furthermore, for each group we obtain a mean diameter $\langle\xi\rangle = \langle\xi(L)\rangle$ after averaging over the members of the seven families with a same value of L within a group. Then, $\langle\xi\rangle$s are averages on 7×7 values of CW diameters. From these sets of 49 CW size measurements we take in addition the standard deviation $\sigma(\xi)$ which measures the 'surface' roughness of the CW. The plot of $L \times \langle\xi\rangle/\rho$ is shown in

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figure 1. It is found that $L=0.032(\langle\xi\rangle/\rho)^{2.75}$ cm with a coefficient of correlation superior to 0.999. Figure 1 shows that all the experimental points fall on a single straight line independently of ρ for L varying over two decades. Since $L\sim$ mass, $D(\mathrm{cw})=2.75$ is the mass-size exponent for cw. This exponent interpolates exactly at half distance between $D(\mathrm{cs})=2.5$ observed for crumpled surfaces [1-3] and $D=3$ for a compact object in the physical tridimensional space. It is interesting to observe that $D(\mathrm{cw})=2.75$ is quite different from the mass-size exponent $1/\nu=5/3$ for a self-avoiding random walk in $d=3$ [5]. Although the crumpling procedure of this experiment seems ill defined, the exponent $D(\mathrm{cw})$ is unaffected by the way of crumpling (with pressure applied, in haste or not). The rugosity of the cw as defined by the standard deviation $\sigma(\xi)$ scales as $\sigma=1.14\rho^{-0.96}\langle\xi\rangle^{0.69}$, i.e. $\sigma(\xi)$ scales as $\sigma(\xi)\sim L^{0.25}$. The surface roughness $\sigma(\xi)$, which is essentially a correlation function, is expected to scale with the uncrumpled size as $\sigma\sim L^{3-D}$ [6], where D is the fractal dimension of the cw. If we use $D=D(\mathrm{cw})=2.75$, the expected exponent for $\sigma(\xi)\times L$ is $3-D=0.25$, exactly as observed.

The electrical resistance R of cw was measured as a function of the size $\langle\xi(L)\rangle$ using a high accuracy digital voltmeter and a stable current source. $R=R(\langle\xi\rangle)$ is an average value on 49 measurements of the electrical resistance for seven cw. To find R the cw were rotated at random without mechanical deformations and the electrodes were adjusted along a particular diameter. The dependence of R with the size $\langle\xi\rangle$ is shown in figure 2. If we study the dependence of $\log(\rho^\alpha R)$ with $\log\langle\xi\rangle$ we obtain the best fit of all the experimental data for $\alpha=1$. The power-law fit to the experimental points is given by $100\rho R=0.25\langle\xi\rangle^{-0.64}$ Ω cm with a coefficient of correlation superior

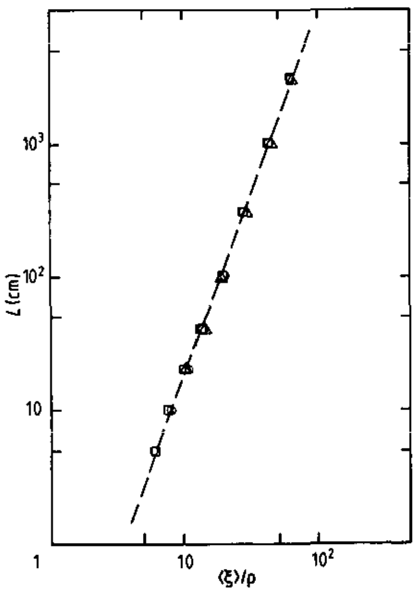

Figure 1. Log-log plot of L against $\langle\xi\rangle/\rho$. $\langle\xi\rangle$ is the average diameter of cws of length L and ρ is the diameter of the wire ($\rho=0.1$ cm (○), $\rho=0.08$ cm (□) and $\rho=0.05$ cm (△)). All the experimental points fall on the straight line $L=0.032(\langle\xi\rangle/\rho)^{2.75}$ (broken line) with a coefficient of correlation superior to 0.999. $D(\mathrm{cw})=2.75$ is the mass-size exponent for cw. The maximal variabilities of the lengths involved in the experiment are $(L/\rho)=60\,000$ and (L-maximal/L-minimal) = 600; see text, second paragraph.

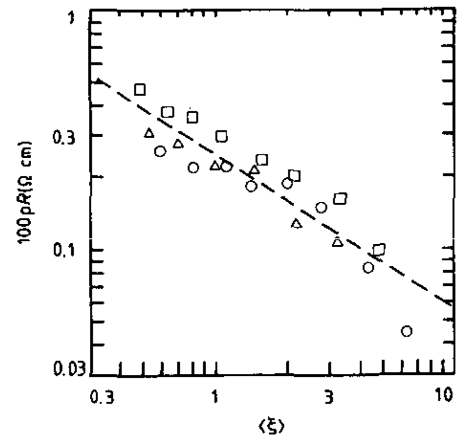

Figure 2. Experimental plot of $100\rho R$ (Ω cm) against $\langle\xi\rangle$ (cm) for wires with diameter $\rho = 0.1$ cm (○), 0.08 cm (□) and 0.05 cm (△). The best fit to these data is obtained with the function $100R = 0.25\langle\xi\rangle^{-0.64}$ Ω cm (broken line). $\zeta = -0.64$ is the electrical resistance exponent for CW; *see text, third paragraph*.

to 0.9. Besides the power-law dependence other fitting functions were used. The power-law fit presented is the best approximation to the experimental points. From the scalings $R \times \langle\xi\rangle$ and $\sigma(\xi) \times \langle\xi\rangle$ we conclude that the product $R \cdot \sigma(\xi)$ is almost independent of the size: $R\sigma(\xi) = 0.003\rho^{-1.96}\langle\xi\rangle^{0.05}$.

The relationship between the relative fluctuations of the electrical resistance, $\sigma(R)/R$, and size, $\sigma(\xi)/\langle\xi\rangle$, is shown in figure 3. In this case several forms of fitting

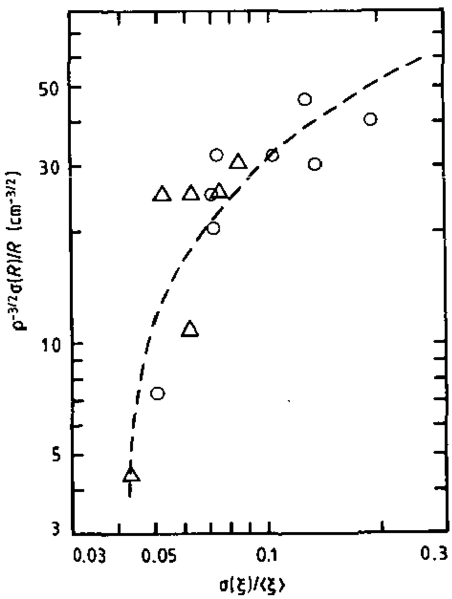

Figure 3. The relationship between the relative fluctuations is the electrical resistance, $\rho^{-3/2}\sigma(R)/R$, and size, $\sigma(\xi)/\langle\xi\rangle$ for $\rho = 0.1$ cm (○) and $\rho = 0.05$ cm (△). The broken curve represents the best fit $\rho^{-3/2}\sigma(R)/R = 128.3[(\sigma(\xi)/\langle\xi\rangle) - 0.042]^{0.48}$; see text, fourth paragraph.

functions were tried. The best adjustment to the experimental points is given by $\rho^{-3/2}\sigma(R)/R = 128.3[(\sigma(\xi)/\langle\xi\rangle) - 0.042]^{0.48}\ \mathrm{cm}^{-3/2}$ (with a coefficient of correlation superior to 0.9) as indicated by the discontinuous line in figure 3. In figure 3 only the experimental points for CW with $\rho = 0.05$ cm and $\rho = 0.1$ cm are exhibited. If the points associated with $\rho = 0.08$ cm are introduced we get the best fit $\rho^{-3/2}\sigma(R)/R = 75.4[(\sigma(\xi)/\langle\xi\rangle) - 0.042]^{0.38}$ and the coefficient of correlation decreases to 0.75. As an additional piece of information about the behaviour of $\sigma(R)/R$ versus $\sigma(\xi)/\langle\xi\rangle$ we observe that a single power-law fitting function presents a further decrease of the coefficient of correlation to circa 0.5.

The electrical resistance R of a physical system of size $\langle\xi\rangle$ scales as $R \sim \langle\xi\rangle^{\zeta}$, where $\zeta = d_W - D$ [7]. The random walk exponent d_W associated with the system of dimension D is defined by $\langle r^2(N)\rangle \sim N^{2/d_W}$ where $\langle r^2\rangle^{1/2}$ is the average distance travelled by a particle diffusing on the structure in the long-time regime $N \gg 1$ [8]. It has been conjectured that $d_W = 2$ for a folded linear chain with a high concentration of bridges incorporated independently of D [9]. This conjecture seems to be observed in our experiment with CW. For CWs the bridges are associated with the lateral contacts between different regions of the wires. Thus two points in contact in the tridimensional space may be quite apart along the wire. If we adopt $d_W = 2$ for our CW and use $D = D(\mathrm{CW}) = 11/4$ as obtained from the experiment (from both plots of $L \times \langle\xi\rangle$ and $\sigma \times L$), the expected electrical resistance exponent ζ would be $\zeta = 2 - 11/4 = -0.75$ which is quite close to the observed value $\zeta_{\mathrm{exp}} = -0.64$ within the experimental errors.

References

[1] Gomes M A F, Braga I J S and Castro C A F 1989 *J. Phys. A: Math. Gen.* **22** 1463–5
[2] Gomes M A F and Vasconcelos G L 1988 *Phys. Rev. Lett.* **60** 237
[3] Gomes M A F 1987 *J. Phys. A: Math. Gen.* **20** L283–4
[4] Gomes M A F and Moura R C 1990 submitted
[5] de Gennes P G 1979 *Scaling Concepts in Polymer Physics* (Ithaca, NY: Cornell University Press)
[6] Gomes M A F, Jyh T I, Ren T I, Rodrigues I M and Furtado C B S 1989 *J. Phys. D: Appl. Phys.* 1217–21
[7] Alexander S and Orbach R 1982 *J. Physique* **43** L625–31
[8] Rammal R and Toulouse G 1983 *J. Physique* **44** L13–22
[9] Helman J S, Coniglio A and Tsallis C 1984 *Phys. Rev. Lett.* **53** 1195–7